Sequences and Series

Algebra 2

Chapter 11

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- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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11.1A Defining and Using Sequences

After this lesson…

• I can use rules to write terms of sequences.

• I can write rules for sequences.

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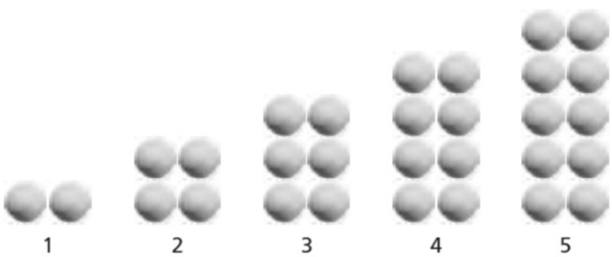
11.1A Defining and Using Sequences

- **Work with a partner.**
- **a.** For each pattern, find the number of objects in each figure. Then find the number of objects in the next figure. Explain your reasoning.



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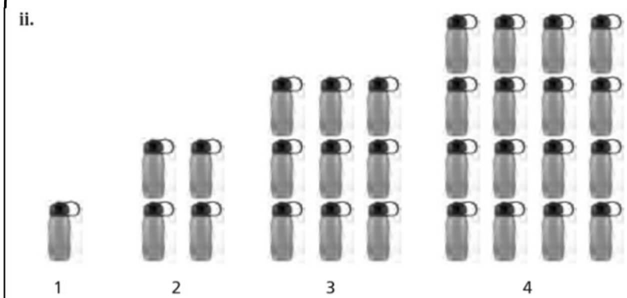

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11.1A Defining and Using Sequences

- Sequence
  - Function whose domain are integers
  - List of numbers that follow a rule

  - 2, 4, 6, 8, 10
    - Finite
  - 2, 4, 6, 8, 10, …
    - Infinite

- Rule
  - $a_n = 2n$

- Domain: (n)
  - Term's location (1st, 2nd, 3rd…)
- Range: (a_n)
  - Term's value (2, 4, 6, 8…)

n is like x, a_n is like y

11.1A Defining and Using Sequences

- Write the first four terms of $a_n = \frac{1}{2}n - 3$

- Try 600#5

$$f(n) = 4^{n-1}$$

$$a_1 = \frac{1}{2}(1) - 3 = -\frac{5}{2}$$
$$a_2 = \frac{1}{2}(2) - 3 = -2$$
$$a_3 = \frac{1}{2}(3) - 3 = -\frac{3}{2}$$
$$a_4 = \frac{1}{2}(4) - 3 = -1$$

$$f(1) = 4^{1-1} = 4^0 = 1$$
$$f(2) = 4^{2-1} = 4^1 = 4$$
$$f(3) = 4^{3-1} = 4^2 = 16$$
$$f(4) = 4^{4-1} = 4^3 = 64$$

11.1A Defining and Using Sequences

- Writing rules for sequences
  - Look for patterns
  - Guess-and-check
  - For fractions, do top and bottom separately
- $\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \ldots$

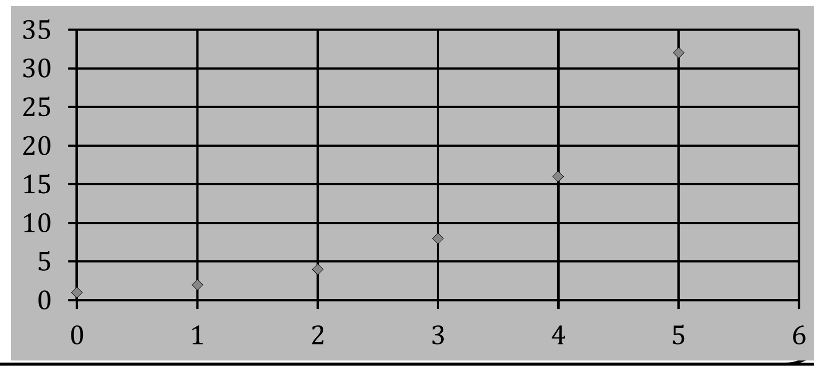
- Try 600#13
  3.1, 3.8, 4.5, 5.2, …

$$\frac{2}{5^1}, \frac{2}{5^2}, \frac{2}{5^3}, \frac{2}{5^4}, \ldots \rightarrow a_n = \frac{2}{5^n}$$

$3.1, 3.1 + (1)0.7, 3.1 + (2)0.7, 3.1 + (3)0.7, \ldots \rightarrow a_n = 3.1 + (n-1)0.7$
$= 3.1 + 0.7n - 0.7 = 0.7n + 2.4$

11.1A Defining and Using Sequences

- To graph
  - n is like x; a_n is like y
  - The graph will be dots
    - Do NOT connect the dots



The n's are integers so there is no values between the integers.

11.1B Defining and Using Series

After this lesson…

- I can write and find sums of series.

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11.1B Defining and Using Series

- Series
  - Sum of a sequence
  - 2, 4, 6, 8, … → sequence
  - $2 + 4 + 6 + 8 + \cdots$ → series

11.1B Defining and Using Series

- Summation Notation (Sigma Notation)
  - Finite

$$2+4+6+8=\sum_{i=1}^{4} 2i$$

Upper limit

Lower limit

  - Infinite

$$2+4+6+8+\cdots=\sum_{i=1}^{\infty} 2i$$

Index of summation (variable)

11.1B Defining and Using Series

- Write as a summation
  - $4 + 8 + 12 + \cdots + 100$

◈ Try 600#25

$$7 + 10 + 13 + 16 + 19$$

a_n = 4n, lower limit = 1, upper limit = 25

$$\sum_{n=1}^{25} 4n$$

#25: a_n = 7 + (n − 1)3 = 3n + 4, lower limit = 1, upper limit = 5

$$\sum_{n=1}^{5} (3n + 4)$$

Note that the index may be any letter.

11.1B Defining and Using Series

- Find the sum of the series

$$\sum_{k=5}^{10} k^2 + 1$$

- Try 600#39

$$\sum_{i=2}^{8} \frac{2}{i}$$

$5^2 + 1 + 6^2 + 1 + 7^2 + 1 + 8^2 +1 + 9^2 + 1 + 10^2 + 1 = 361$

#39

$$\frac{2}{2}+\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\frac{2}{6}+\frac{2}{7}+\frac{2}{8}=\frac{481}{140}\approx 3.436$$

11.1B Defining and Using Series

- Some shortcut formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

11.1B Defining and Using Series

- Find the sum of the series

$$\sum_{k=1}^{10} 3k^2 + 2$$

- Try 600#43

$$\sum_{i=10}^{25} i$$

$$3\frac{n(n+1)(2n+1)}{6} + 2n = 3\frac{10(10+1)(2(10)+1)}{6} + 2(10) = 1175$$

#43

$$\sum_{i=10}^{25} i = \sum_{i=1}^{25} i - \sum_{i=1}^{9} i$$

$$= \frac{n(n+1)}{2} - \frac{n(n+1)}{2} = \frac{25(25+1)}{2} - \frac{9(9+1)}{2} = 325 - 45 = 280$$

11.2 Analyzing Arithmetic Sequences and Series

After this lesson…

- I can identify arithmetic sequences.
- I can write rules for arithmetic sequences.
- I can find sums of finite arithmetic series.

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11.2 Analyzing Arithmetic Sequences and Series

- **Work with a partner.** When German mathematician Carl Friedrich Gauss (1777–1855) was young, one of his teachers asked him to find the sum of the whole numbers from 1 through 100. To the astonishment of his teacher, Gauss came up with the answer after only a few moments. Here is what Gauss did:

$$\begin{array}{ccccccc} 1 & + & 2 & + & 3 & + \cdots + & 100 \\ 100 & + & 99 & + & 98 & + \cdots + & 1 \\ \hline 101 & + & 101 & + & 101 & + \cdots + & 101 \end{array}$$

- $\frac{100 \times 101}{2} = 5050$
- **a.** Explain Gauss's thought process. Then write a formula for the sum S_n of the first n terms of an arithmetic sequence.

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$$S_n = \frac{n}{2}(a_1 + a_n)$$

11.2 Analyzing Arithmetic Sequences and Series

- Arithmetic Sequences
  - Common difference (d) between successive terms
    - Add the same number each time
  - 3, 6, 9, 12, 15, …
    - $d = 3$

- Is it arithmetic?
  - −10, −6, −2, 0, 2, 6, 10, …

  - Try 608#1
    1, −1, −3, −5, −7, …

No
Yes, d = −2

11.2 Analyzing Arithmetic Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 + (n-1)d$
  - Linear
- Write a rule for the n^{th} term
  - 32, 47, 62, 77, …

- Try 608#13
  51, 48, 45, 42, …

$$d = 15$$
$$a_n = 32 + (n-1)15 = 32 + 15n - 15 \rightarrow a_n = 17 + 15n$$

#13

$$d = -3, a_1 = 51$$
$$a_n = a_1 + (n-1)d$$
$$a_n = 51 + (n-1)(-3) = 51 - 3n + 3 \rightarrow a_n = -3n + 54$$

11.2 Analyzing Arithmetic Sequences and Series

- One term of an arithmetic sequence is $a_8 = 50$. The common difference is 0.25. Write the rule for the n^{th} term.

- Try 608#21

$$a_{11} = 43, d = 5$$

$$a_n = a_1 + (n-1)d$$

$$50 = a_1 + (8-1)0.25 \rightarrow 50 = a_1 + 1.75 \rightarrow 48.25 = a_1$$

$$a_n = 48.25 + (n-1)0.25 \rightarrow a_n = 48.25 + 0.25n - 0.25 \rightarrow a_n = 48 + 0.25n$$

#21

$$a_n = a_1 + (n-1)d$$

$$43 = a_1 + (11-1)5 \rightarrow 43 = a_1 + 50 \rightarrow -7 = a_1$$

$$a_n = -7 + (n-1)5 \rightarrow a_n = -7 + 5n - 5 \rightarrow a_n = 5n - 12$$

11.2 Analyzing Arithmetic Sequences and Series

- Two terms of an arithmetic sequence are $a_5 = 10$ and $a_{30} = 110$. Write a rule for the n^{th} term.

$a_n = a_1 + (n - 1)d$

$10 = a_1 + (5 - 1)d$ → $10 = a_1 + 4d$

$110 = a_1 + (30 - 1)d$ → $110 = a_1 + 29d$

Linear combination

$-10 = -a_1 - 4d$

$\underline{110 = a_1 + 29d}$

$100 = 25\,d$

$d = 4$

Substitute

$10 = a_1 + 4d$ → $10 = a_1 + 4(4)$ → $a_1 = -6$

Rule

$a_n = -6 + (n - 1)4$ → $a_n = -6 + 4n - 4$ → $a_n = 4n - 10$

11.2 Analyzing Arithmetic Sequences and Series

- Try 608#33
  Two terms of an arithmetic sequence are $a_8 = 12$ and $a_{16} = 22$. Write a rule for the n^{th} term.

$a_n = a_1 + (n – 1)d$
$12 = a_1 + (8 – 1)d \rightarrow 12 = a_1 + 7d$
$22 = a_1 + (16 – 1)d \rightarrow 22 = a_1 + 15d$
Linear combination
$-12 = -a_1 – 7d$
$\underline{22 = a_1 + 15d}$
$10 = 8d$
$d = 5/4$

Substitute
$12 = a_1 + 7d \rightarrow 12 = a_1 + 7(5/4) \rightarrow 12 = a_1 + 35/4 \rightarrow a_1 = 13/4$

Rule
$a_n = 13/4 + (n – 1)5/4 \rightarrow a_n = 13/4 + 5n/4 – 5/4 \rightarrow a_n = (5/4)n + 2$

11.2 Analyzing Arithmetic Sequences and Series

- Sum of a finite arithmetic series
  - 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
  - Rewrite
    - 1 + 2 + 3 + 4 + 5
    - <u>10 + 9 + 8 + 7 + 6</u>
    - 11+11 +11+11 +11 = 5(11) = 55
  - Formula
    - $S_n = n\left(\frac{a_1+a_n}{2}\right)$

From example:
First and last (a_1 + a_n) = 11
10 numbers but only half as many pairs (n/2)

11.2 Analyzing Arithmetic Sequences and Series

- Consider the arithmetic series
  - $20 + 18 + 16 + 14 + \cdots$
- Find the sum of the first 25 terms.

- Try 609#41

$$\sum_{i=1}^{20} (2i - 3)$$

$$a_{25} = 20 + (25 - 1)(-2) = -28$$

$$S_{25} = 25\left(\frac{20 + (-28)}{2}\right) = -100$$

#41: This is arithmetic because the rule is linear

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S_{20} = 20\left(\frac{(2(1) - 3) + (2(20) - 3)}{2}\right) = 20(18) = 360$$

11.2 Analyzing Arithmetic Sequences and Series

- You put money in a jar at the end of each week. The first week you put \$2 in the jar, and each subsequent week you put \$2 more than the previous week in the jar.
- **a.** Write a rule for the amount of money you put in the jar at the end of the *n*th week.

- **b.** How much money is in the jar after 9 weeks?

25

2, 4, 6, 8, …
$a_n = 2n$

$S_n = \frac{n(a_1+a_n)}{2} = \frac{9(2+18)}{2}$ =\$90

11.3 Analyzing Geometric Sequences and Series

After this lesson…

- I can identify geometric sequences.
- I can write rules for geometric sequences.
- I can find sums of finite geometric series.

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11.3 Analyzing Geometric Sequences and Series

- Geometric Sequence
  - Created by multiplying by a common ratio (r)

- Are these geometric sequences?
  - 1, 2, 6, 24, 120, …

  - Try 616#1
    96, 48, 24, 12, 6, …

No
Yes r = 1/2

11.3 Analyzing Geometric Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 \cdot r^{n-1}$
  - Exponential
- Write a rule for the n^{th} term and find a_8.
  - 5, 2, 0.8, 0.32, …

- Try 616#13
  112, 56, 28, 14, …

$$r = \frac{2}{5}$$
$$a_n = 5\left(\frac{2}{5}\right)^{n-1}$$
$$a\_8 = 5\left(\frac{2}{5}\right)^{7} = 0.008192$$

#13

$$a_1 = 112; r = \frac{1}{2}$$
$$a_n = a_1 r^{n-1}$$
$$a_n = 112\left(\frac{1}{2}\right)^{n-1}$$

11.3 Analyzing Geometric Sequences and Series

- One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the n^{th} term.

- Try 616#23
  One term of a geometric sequence is $a_4 = -192$ and $r = 4$. Write the rule for the n^{th} term.

$$a_4 = 3 = a_1 3^{4-1} \rightarrow 3 = a_1\, 27 \rightarrow a_1 = \frac{1}{9}$$

$$a_n = \left(\frac{1}{9}\right) 3^{n-1}$$

#23

$$a_n = a_1 r^{n-1}$$

$$-192 = a_1(4)^{4-1} \rightarrow -192 = a_1(64) \rightarrow \ a_1 = -3$$

$$a_n = -3(4)^{n-1}$$

11.3 Analyzing Geometric Sequences and Series

- If two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$, write rule for the n^{th} term.

$$a_2 = -4 = a_1\, r^{2-1} \rightarrow -4 = a_1\, r$$
$$a_6 = -1024 = a_1\, r^{6-1} \rightarrow -1024 = a_1\, r^5$$

Solve first for a_1: $a_1 = -\frac{4}{r}$

Plug into second: $-1024 = \left(-\frac{4}{r}\right) r^5 \rightarrow -1024 = -\frac{4r^5}{r} \rightarrow -1024 = -4r^4 \rightarrow 256 = r^4 \rightarrow r = 4$

Plug back into first: $a_1 = -\frac{4}{4} \rightarrow a_1 = -1$

Write rule: $a_n = -1 \cdot 4^{n-1}$

11.3 Analyzing Geometric Sequences and Series

- Try 616#35
  If two terms of a geometric sequence are $a_2 = -72$ and $a_6 = -\frac{1}{18}$, write rule for the n^{th} term.

$$a_2 = -72 = a_1\, r^{2-1} \rightarrow -72 = a_1\, r$$
$$a_6 = -\frac{1}{18} = a_1\, r^{6-1} \rightarrow -\frac{1}{18} = a_1\, r^5$$

Solve first for a_1: $a_1 = -\frac{72}{r}$

Plug into second: $-\frac{1}{18} = \left(-\frac{72}{r}\right) r^5 \rightarrow -\frac{1}{18} = -\frac{72r^5}{r} \rightarrow -\frac{1}{18} = -72r^4 \rightarrow \frac{1}{1296} = r^4 \rightarrow r = \frac{1}{6}$

Plug back into first: $a_1 = -\frac{72}{1/6} \rightarrow a_1 = -432$

Write rule: $a_n = -432 \cdot \left(\frac{1}{6}\right)^{n-1}$

11.3 Analyzing Geometric Sequences and Series

- Sum of geometric series
  - $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
- Find the sum of the first 10 terms of
  - 4 + 2 + 1 + ½ + ⋯

- Try 617#44

$$\sum_{i=1}^{8} 5\left(\frac{1}{3}\right)^{i-1}$$

$$r = \frac{1}{2}, a_1 = 4$$

$$S_{10} = 4\left(\frac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}}\right) = 4\left(\frac{.99902}{.5}\right) = 7.992 = \frac{1023}{128}$$

#44: geometric because exponential

$$a_n = a_1 r^{n-1}$$

$$a_1 = 5, r = \frac{1}{3}$$

$$S_8 = a_1\left(\frac{1-r^n}{1-r}\right) = 5\left(\frac{1-\left(\frac{1}{3}\right)^{8}}{1-\frac{1}{3}}\right) = \frac{16400}{2187} \approx 7.499$$

11.3 Analyzing Geometric Sequences and Series

- You tell the Gospel to your friends. Four of your friends tell the Gospel to their friends, then four of each of their friends tells the Gospel, and so on. Find the total number of people who told the Gospel to others after the eighth round.

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$$r = 4, a_1 = 4$$

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_8 = 4\left(\frac{1 - 4^8}{1 - 4}\right) = 87380\ people$$

11.4 Finding Sums of Infinite Geometric Series

After this lesson…

- I can find partial sums of infinite geometric series.
- I can find sums of infinite geometric series.
- I can solve real-life problems using infinite geometric series.

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11.4 Finding Sums of Infinite Geometric Series

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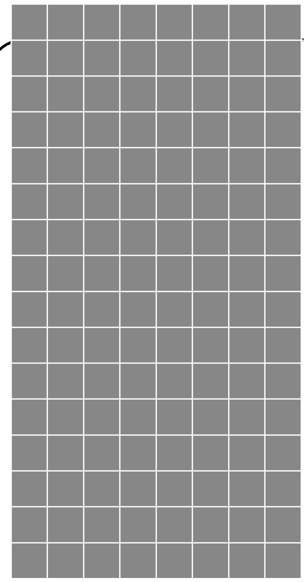
11.4 Finding Sums of Infinite Geometric Series

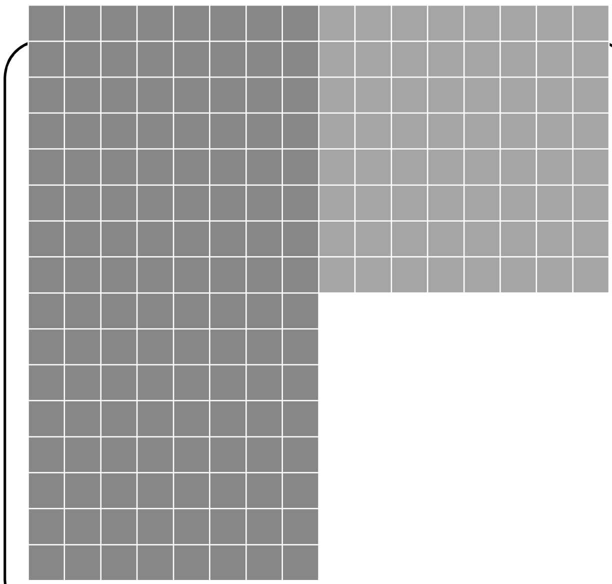
$$\frac{1}{2} = 0.5$$



11.4 Finding Sums of Infinite Geometric Series

$$\frac{1}{2} = 0.5$$

$$\frac{1}{2} + \frac{1}{4} = 0.75$$



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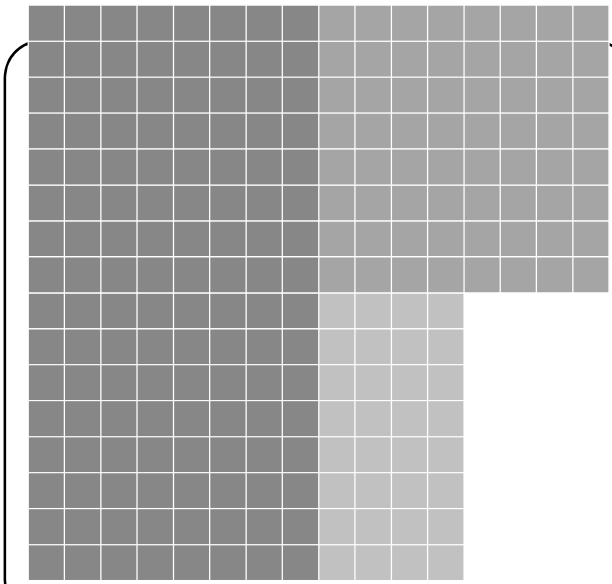
11.4 Finding Sums of Infinite Geometric Series

$$\frac{1}{2} = 0.5$$

$$\frac{1}{2} + \frac{1}{4} = 0.75$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$$



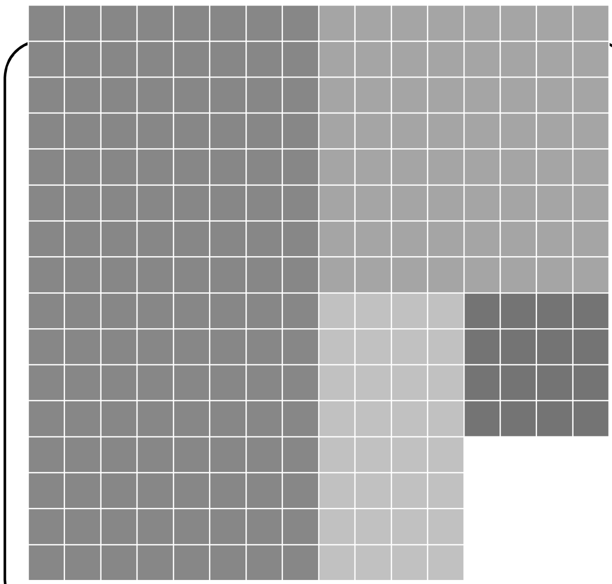
11.4 Finding Sums of Infinite Geometric Series

$$\frac{1}{2} = 0.5$$

$$\frac{1}{2} + \frac{1}{4} = 0.75$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375$$



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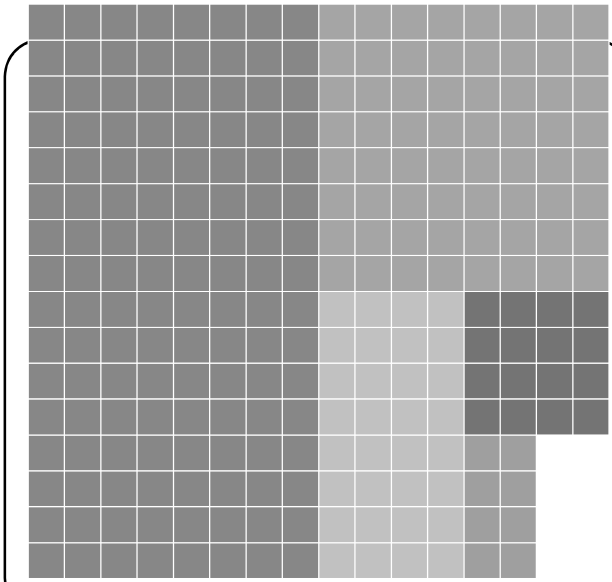
11.4 Finding Sums of Infinite Geometric Series

$$\frac{1}{2} = 0.5$$
$$\frac{1}{2} + \frac{1}{4} = 0.75$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = 0.96875$$



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11.4 Finding Sums of Infinite Geometric Series

$$\frac{1}{2} = 0.5$$

$$\frac{1}{2} + \frac{1}{4} = 0.75$$

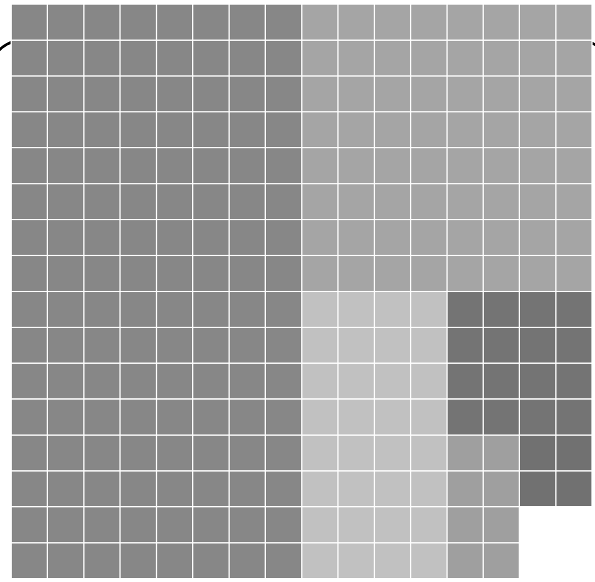
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = 0.96875$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = 0.984375$$

What happens if the pattern continues forever?



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Eventually the entire square is filled in

- Find the partial sums for n = 1, 2, 3, 4, 5 and describe what happens to S_n as n increases.
  $\frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}+\frac{1}{80}+\cdots$

- Try 623#3
  $4+\frac{12}{5}+\frac{36}{25}+\frac{108}{125}+\frac{324}{625}+\cdots$

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$$S_1=\frac{1}{5}=0.2$$
$$S_2=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}=0.3$$
$$S_3=\frac{1}{5}+\frac{1}{10}+\frac{1}{20}=\frac{7}{20}=0.35$$
$$S_4=\frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}=\frac{15}{40}=0.375$$
$$S_5=\frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}+\frac{1}{80}=\frac{31}{80}=0.3875$$

Approaching 4

#3

$$S_1=4$$
$$S_2=4+\frac{12}{5}=\frac{32}{5}=6.4$$
$$S_3=4+\frac{12}{5}+\frac{36}{25}=\frac{196}{25}=7.84$$
$$S_4=4+\frac{12}{5}+\frac{36}{25}+\frac{108}{125}=\frac{1088}{125}=8.704$$
$$S_5=4+\frac{12}{5}+\frac{36}{25}+\frac{108}{125}+\frac{324}{625}=\frac{5764}{625}=9.2224$$

Approaches 10

11.4 Finding Sums of Infinite Geometric Series

- Sum of an infinite geometric series
  - $S = \frac{a_1}{1-r}$

- $| r | < 1$
  - If $| r | > 1$, then no sum (∞)

11.4 Finding Sums of Infinite Geometric Series

- Find the sum

$$\sum_{i=1}^{\infty} 2(0.1)^{i-1}$$

- Try 623#9

$$2 + \frac{6}{4} + \frac{18}{16} + \frac{54}{64} + \cdots$$

$$a_1 = 2(0.1)^{1-1} = 2, r = 0.1$$

$$S = \frac{2}{1-0.1} = \frac{2}{0.9} = \frac{20}{9}$$

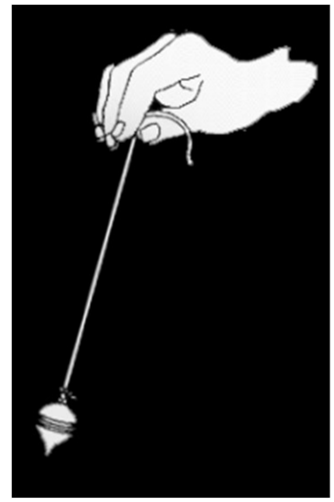
#9

$$a_1 = 2, r = \frac{3}{4}$$

$$S = \frac{2}{1-\frac{3}{4}} = \frac{2}{\frac{1}{4}} = 8$$

11.4 Finding Sums of Infinite Geometric Series

- A pendulum that is released and swings freely travels 100 centimeters on the first swing. On each successive swing, the pendulum travels 96% of the distance of the previous swing. What is the total distance the pendulum travels?



$$a_1 = 100, r = 0.96$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{100}{1-0.96} = 2500\ cm = 25\ m$$

11.4 Finding Sums of Infinite Geometric Series

- Write $0.27272727\ldots$ as a fraction.

Write the repeating unit as a sum of fractions

$$\frac{27}{100}+\frac{27}{10000}+\frac{27}{1000000}+\ \ldots$$

$$a_1=\frac{27}{100}, r=\frac{1}{100}$$

$$S=\frac{\frac{27}{100}}{1-\left(\frac{1}{100}\right)}=\frac{\frac{27}{100}}{\frac{99}{100}}=\frac{27}{99}=\frac{3}{11}$$

11.4 Finding Sums of Infinite Geometric Series

- Try 623#21
  Write 32.323232… as a fraction.

#21

$$32 + \frac{32}{100} + \frac{32}{10000} + \frac{32}{1000000} + \cdots$$

$$a_1 = 32, r = \frac{1}{100}$$

$$S_\infty = \frac{a_1}{1-r} = \frac{32}{1 - \frac{1}{100}}$$

$$= \frac{32}{99/100}$$

$$= \frac{3200}{99}$$

11.5 Using Recursive Rules with Sequences

After this lesson…

- I can write terms of recursively defi ned sequences.
- I can write recursive rules for sequences.
- I can translate between recursive rules and explicit rules.
- I can use recursive rules to solve real-life problems.

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11.5 Using Recursive Rules with Sequences

- Explicit Rule
  - Gives the n^{th} term directly
  - $a_n = 2 + 4n$

- Recursive Rule
  - Each term is found by knowing the previous term
  - $a_1 = 6; a_n = a_{n-1} + 4$

Both these rules give the same sequence

11.5 Using Recursive Rules with Sequences

- Write the first 5 terms
  - $a_1 = 3, a_n = 2a_{n-1} - 1$

◈ Try 631#5
$a_1 = 2; a_n = (a_{n-1})^2 + 1$

$$a_1 = 3,$$
$$a_2 = 2(3) - 1 = 5,$$
$$a_3 = 2(5) - 1 = 9,$$
$$a_4 = 2(9) - 1 = 17,$$
$$a_5 = 2(17) - 1 = 33$$

#5

$$a_1 = 2,$$
$$a_2 = (2)^2 + 1 = 5,$$
$$a_3 = (5)^2 + 1 = 26,$$
$$a_4 = (26)^2 + 1 = 677,$$
$$a_5 = (677)^2 + 1 = 458330$$

11.5 Using Recursive Rules with Sequences

- Special Recursive Rules
  - Arithmetic Sequence
    - $a_n = a_{n-1} + d, a_1 = a_1$
  - Geometric Sequence
    - $a_n = r \cdot a_{n-1}, a_1 = a_1$

11.5 Using Recursive Rules with Sequences

- Write the rules for the arithmetic sequence where a_1 = 15 and d = 5.
  - Explicit
  - Recursive

Explicit: $a_n = 15 + (n - 1)5 \quad a_n = 5n + 10$

Recursive: $a_1 = 15, a_n = a_{n-1} + 5$

11.5 Using Recursive Rules with Sequences

- Write the rule for the geometric sequence where a_1 = 4 and r = 0.2
  - Explicit
  - Recursive

Explicit: $a_n = 4(0.2)^{n-1}$

Recursive: $a_1 = 4, a_n = 0.2a_{n-1}$

11.5 Using Recursive Rules with Sequences

- Write a recursive rule for
  - 1, 1, 4, 10, 28, 76, …

◈ Try 631#13

$44, 11, \frac{11}{4}, \frac{11}{16}, \frac{11}{64}, \ldots$

$$a_1 = 1, a_2 = 1, a_n = 2(a_{n-2} + a_{n-1})$$

#13: geometric, multiplying by 1/4

$$a_1 = 1, a_n = \frac{1}{4}a_{n-1}$$

11.5 Using Recursive Rules with Sequences

- Write a recursive rule for
$$a_n = 30 - 5n$$

- Try 631#29
$$a_n = 12(11)^{n-1}$$

This is arithmetic in the form where d = −5 and $a_1 = 30 - 5(1) = 25$

$$a_n = a_{n-1} + d$$
$$a_n = a_{n-1} - 5, a_1 = 25$$

This is geometric with $a_n = a_1 \cdot r^{n-1}$ where r = 11 and $a_1 = 12$

$$a_n = r \cdot a_{n-1}$$
$$a_n = 11 \cdot a_{n-1}, a_1 = 12$$

11.5 Using Recursive Rules with Sequences

- Write an explicit rule for each sequence.

$$a_1 = 7, a_n = a_{n-1} + 4$$

- Try 631#39

$$a_1 = -2;\ a_n = 3a_{n-1}$$

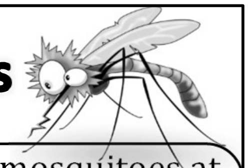
This is arithmetic because it is adding with form $a_n = a_{n-1} + d$. d = 4

$$a_n = a_1 + (n-1)d$$
$$a_n = 7 + (n-1)4$$
$$a_n = 7 + 4n - 4$$
$$a_n = 4n + 3$$

This is geometric because it is multiplying from $a_n = r \cdot a_{n-1}$. r = 3

$$a_n = a_1 \cdot r^{n-1}$$
$$a_n = -2(3)^{n-1}$$

11.5 Using Recursive Rules with Sequences



- A controlled laboratory contains about 500 mosquitoes. Each day, 100 new mosquitoes hatch, but the population declines 85% due to a pesticide and natural causes.
- **a.** Write a recursive rule for the number a_n of mosquitoes at the start of the n^{th} day.

- **b.** Find the number of mosquitoes at the start of the fourth day.

- **c.** Describe what happens to the population of mosquitoes over time.

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If lose 85%, then still have 15%

$$a_1 = 500, a_n = 0.15a_{n-1} + 100$$

$$a_1 = 500$$
$$a_2 = 0.15(500) + 100 = 175$$
$$a_3 = 0.15(175) + 100 = 126.25$$
$$a_4 = 0.15(126.25) + 100 = 118.9375$$

about 119 mosquitoes

Find lots of points by using calculator with .15·ans+100
Approaches 118 mosquitoes

11.5 Using Recursive Rules with Sequences

- Try 632#53
  You borrow $2000 to travel. The loan has a 9% annual interest rate that is compounded monthly for 2 years. The monthly payment is $91.37.

a. Find the balance after the fifth payment.

b. Find the amount of the last payment.

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a. Monthly interest rate is 0.09/12=0.0075

$$a_0 = 2000;\ a_n = a_{n-1} + 0.0075a_{n-1} - 91.37 = 1.0075a_{n-1} - 91.37$$
$$a_1 = 1.0075(2000) - 91.37 = 1923.63$$
$$a_2 = 1.0075(1923.63) - 91.37 = 1846.69$$
$$a_3 = 1.0075(1846.69) - 91.37 = 1769.17$$
$$a_4 = 1.0075(1769.17) - 91.37 = 1691.07$$
$$a_5 = 1.0075(1691.07) - 91.37 = 1612.38$$

b. Use a calculator and enter 1.0075(2000)-91.37 and press enter. Then enter 1.0075(ans)-91.37 and press enter repeatedly until reaching 0.
$90.68